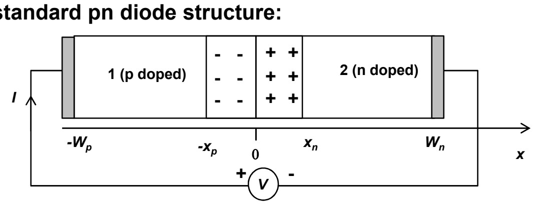
Photodetectors and Solar Cells

In this lecture you will learn:

- **Photodiodes**
- **Avalanche Photodiodes**
- **Solar Cells**
- **Fundamentals Limits on Solar Energy Conversion**
- **Practical Solar Cells**

Photodiodes

Consider the standard pn diode structure:

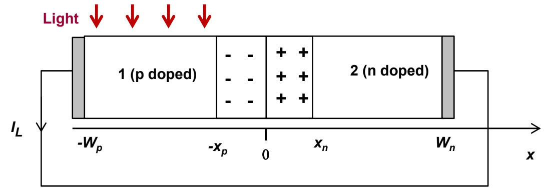


$$I = I_o\left(e^{\frac{qV}{KT}} - 1\right)$$

The pn diode can be used as a photodiode

Photogeneration in the Quasineutral Region

Consider an unbiased photodiode with light shining on the p quasineutral region:



We need to compute the current I_L in the external circuit

Start from:

$$\cancelto{0}{\frac{\partial n}{\partial t}} - \frac{1}{q}\frac{\partial}{\partial x}J_e(x) = G_e(x) - R_e(x) + G_L(x)$$

← **Generation term due to light**

Assumptions:

$$J_e(x) = qD_{e1}\frac{\partial n(x)}{\partial x}$$ { **minority carrier current by diffusion only**

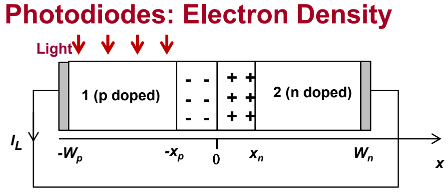
$$G_e(x) - R_e(x) = -\frac{\left(n(x) - n_{po}\right)}{\tau_{e1}}$$

$$G_L(x) = \text{constant} = G_L$$

Shockley's Equations for Photodiodes: Electron Density

We get for the minority carrier density:

$$\frac{\partial^2 n(x)}{\partial x^2} = \frac{\left(n(x) - n_{po}\right)}{L_{e1}^2} - \frac{G_L}{D_{e1}}$$



Boundary conditions:

$$n\left(x = -x_p\right) = n_{po}e^{qV/KT} = n_{po}$$ ← { **At depletion region edge**

$$n\left(x = -W_p\right) = n_{po}$$ ← { **At the left metal contact**

Solution:

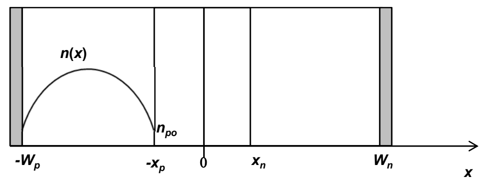
$$n(x) = n_{po} + G_L\tau_{e1} - G_L\tau_{e1}\left[\frac{\sinh\left(\frac{W_p + x}{L_{e1}}\right) - \sinh\left(\frac{x + x_p}{L_{e1}}\right)}{\sinh\left(\frac{W_p - x_p}{L_{e1}}\right)}\right]$$

Solution in the short base limit is:

$$n(x) = n_{po} + \frac{G_L}{2D_{e1}}\left(\frac{W_p - x_p}{2}\right)^2 - \frac{G_L}{2D_{e1}}\left(x + \frac{W_p + x_p}{2}\right)^2$$ { $L_{e1} >> \left(W_p - x_p\right)$

Shockley's Equations for Photodiodes: Electron and Hole Currents

$$n(x) = n_{po} + G_L \tau_{e1} - G_L \tau_{e1} \left[\frac{\sinh\left(\frac{W_p + x}{L_{e1}}\right) - \sinh\left(\frac{x + x_p}{L_{e1}}\right)}{\sinh\left(\frac{W_p - x_p}{L_{e1}}\right)} \right]$$



Electron Current:

Electron (minority carrier) current flows by diffusion:

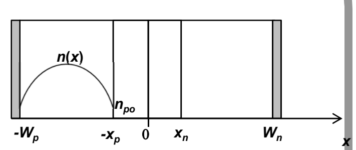
$$J_e(x) = q D_{e1} \frac{\partial n}{\partial x} = -q G_L L_{e1} \left[\frac{\cosh\left(\frac{W_p + x}{L_{e1}}\right) - \cosh\left(\frac{x + x_p}{L_{e1}}\right)}{\sinh\left(\frac{W_p - x_p}{L_{e1}}\right)} \right]$$

$$= -q G_L \left(x + \frac{W_p + x_p}{2} \right) \qquad \{ \text{Short base limit}$$

Shockley's Equations for Photodiodes: Electron and Hole Currents

Total Current:

Total current is due to the electrons that reach the depletion region edge

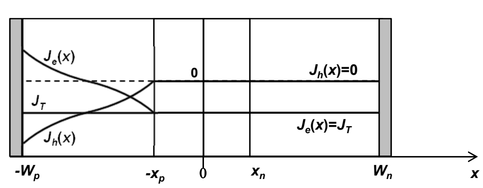


$$J_T = J_e(-x_p) = -q G_L L_{e1} \left[\frac{\cosh\left(\frac{W_p - x_p}{L_{e1}}\right) - 1}{\sinh\left(\frac{W_p - x_p}{L_{e1}}\right)} \right]$$

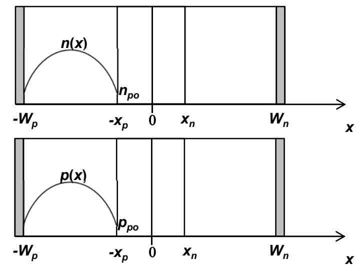
$$= -q G_L \left(\frac{W_p - x_p}{2} \right) \qquad \{ \text{Short base limit}$$

Hole Current:

$$J_h(x) = J_T - J_e(x)$$



Shockley's Equations for Photodiodes: Hole Currents



Quasineutrality implies:

$$\Delta p(x) = \Delta n(x)$$

Hole Diffusion Current:

$$J_{h,diff}(x) = -q\, D_{h1} \frac{\partial \Delta p}{\partial x}$$

$$= -q\, D_{h1} \frac{\partial \Delta n}{\partial x} = -\frac{D_{h1}}{D_{e1}} J_e(x)$$

Hole Drift Current:

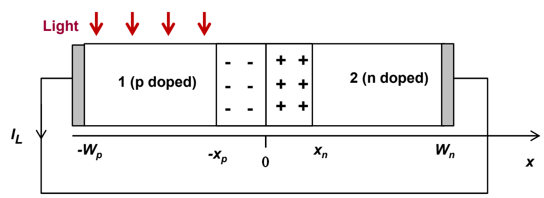
$$J_h(x) = J_T - J_e(x)$$

$$\Rightarrow J_{h,drift}(x) + J_{h,diff}(x) = J_T - J_e(x)$$

$$\Rightarrow J_{h,drift}(x) = J_T - J_e(x) - J_{h,diff}(x)$$

$$\Rightarrow J_{h,drift}(x) = J_T - \left[1 - \frac{D_{h1}}{D_{e1}}\right] J_e(x)$$

External Circuit Current



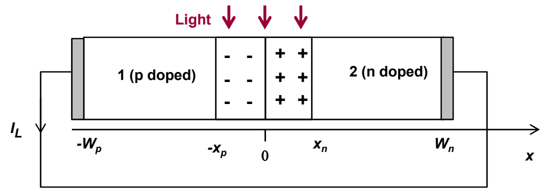
$$I_L = -AJ_T$$

$$I_L = qG_L A L_{e1} \left[\frac{\cosh\left(\frac{W_p - x_p}{L_{e1}}\right) - 1}{\sinh\left(\frac{W_p - x_p}{L_{e1}}\right)} \right]$$

$$= qG_L A \left(\frac{W_p - x_p}{2}\right) \qquad \{\text{Short base limit}$$

Even if one ignores recombination inside the quasineutral region (short base limit), the circuit current corresponds to only half the total number of electron-hole pairs generated by light inside the detector (why?)

Photogeneration in the Depletion Region



Electron Current:

$$-\frac{1}{q}\frac{\partial J_e(x)}{\partial x} = G_L$$

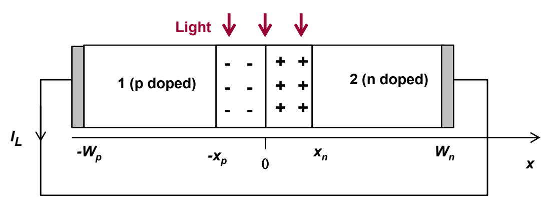
$$\Rightarrow J_e(-x_p) - J_e(x_n) = q\int_{-x_p}^{x_n} G_L \, dx = qG_L(x_n + x_p)$$

Hole Current:

$$\frac{1}{q}\frac{\partial J_h(x)}{\partial x} = G_L$$

$$\Rightarrow J_h(x_n) - J_h(-x_p) = q\int_{-x_p}^{x_n} G_L \, dx = qG_L(x_n + x_p)$$

Electron, Hole, and Total Currents



Electron and Hole Current:

The electric field inside the depletion region sweeps the electrons towards the n-side and the holes towards the p-side. Consequently, it must be that:

$$J_e(-x_p) = 0$$
$$J_h(x_n) = 0$$

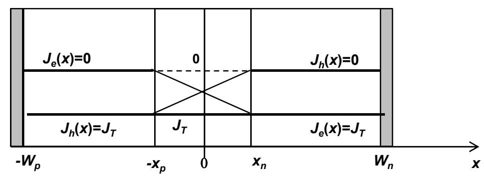
Therefore,

$$J_h(-x_p) = J_e(x_n) = -qG_L(x_n + x_p)$$

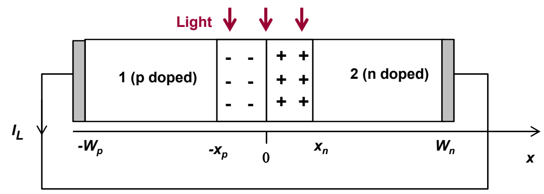
Total Current:

$$J_T = J_e(x_n) + J_h(x_n) = J_e(-x_p) + J_h(-x_p)$$
$$= -qG_L(x_n + x_p)$$

Electron, Hole, and Total Currents



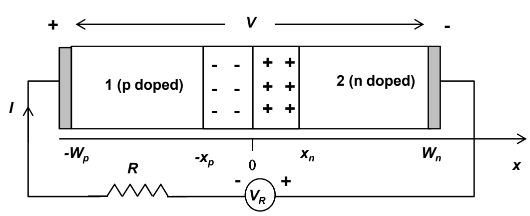
External Circuit Current



$$I_L = -AJ_T = qG_L\left(x_n + x_p\right)$$

The circuit current corresponds to the total number of electron-hole pairs generated by light inside the detector!

Photodetector Circuits: Electrical Characteristics



Photodetectors are operated in reverse bias (why?)

The circuit current I has two components:

i) The current due to the biased pn junction given as:

$$I_o\left(e^{qV/KT} - 1\right)$$

ii) The current I_L due to photogeneration

These two components can be added together (why?) to give the total current:

$$I = I_o\left(e^{qV/KT} - 1\right) - I_L$$

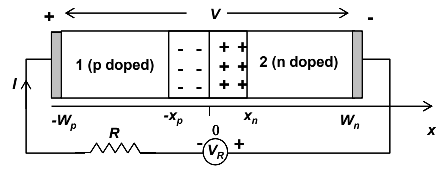
Photodetector Circuits: Electrical Characteristics

We had:

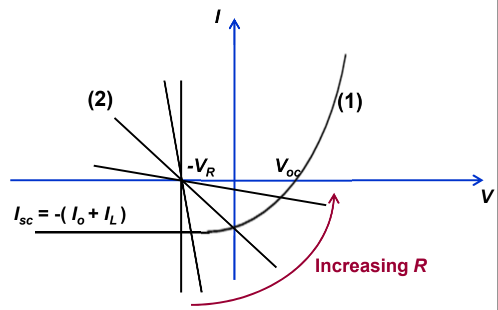
$$I = I_o\left(e^{qV/KT} - 1\right) - I_L \qquad (1)$$

Kirchhoff's Voltage Law gives:

$$IR + V_R + V = 0 \qquad (2)$$



The solution of these two equations gives the circuit current and the junction voltage



Open circuit voltage ($R = \infty$):

$$V = V_{oc} = \frac{KT}{q}\ln\left(\frac{I_L}{I_o} + 1\right)$$

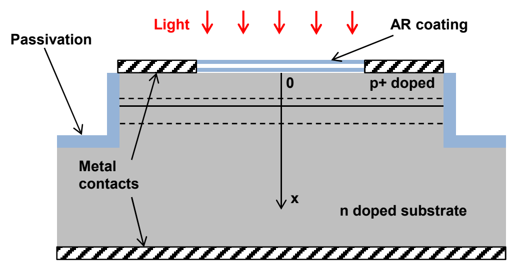
Short circuit current ($R = 0$): :

$$I = I_{sc} = -\left(I_o + I_L\right)$$

I_L → Current due to photogeneration

I_o → Current due to thermal generation

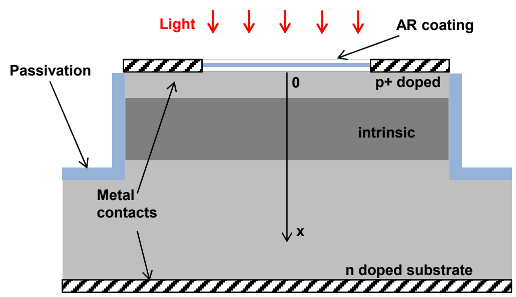
Common Photodetector Structures



Front side illuminated homojunction photodetector

$$I(x) = I_o e^{-\alpha x}$$

$$G_L(x) = R_\uparrow - R_\downarrow = \alpha\frac{I(x)}{\hbar\omega} = \alpha\frac{I_o e^{-\alpha x}}{\hbar\omega}$$

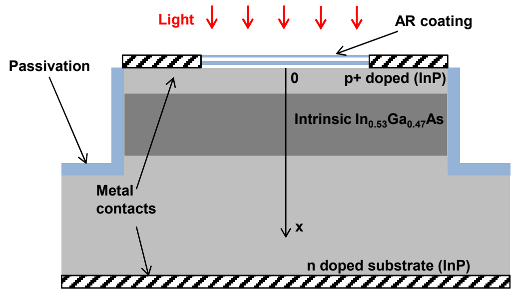


Front side illuminated homojunction pin photodetector

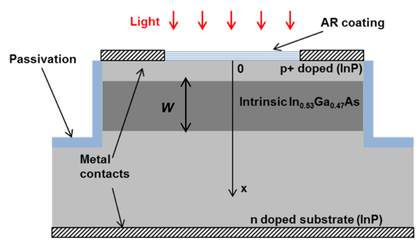
Common Photodetector Structures

Front side illuminated heterojunction pin photodetector (used for 1550 nm fiber optic communications)

Photogeneration only in the intrinsic region!



Figures of Merit of Photodetectors



Responsivity (units: Amps/Watt):

$$R = \frac{I_L}{P_{inc}}$$

External Quantum Efficiency:

$$\eta_{ext} = \frac{I_L/q}{P_{inc}/\hbar\omega} = \frac{\hbar\omega}{q}\frac{I_L}{P_{inc}} = \frac{\hbar\omega}{q}R$$

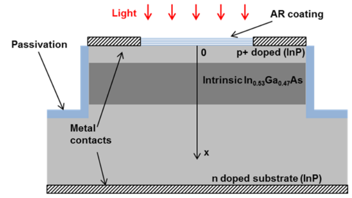
External quantum efficiency measures the fraction of photons incident on the photodetector that ended up producing an electron in the external circuit

Internal Quantum Efficiency:

$$\eta_{int} = \frac{I_L/q}{P_{abs}/\hbar\omega} = \frac{\hbar\omega}{q}\frac{I_L}{P_{abs}}$$

Internal quantum efficiency measures the fraction of photons that caused photogeneration and also ended up producing an electron in the external circuit

Figures of Merit of Photodetectors



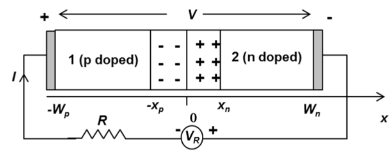
Dark Current:

The dark current of a photodetector is the current present even if there no light and limits the detector sensitivity

When V_R=0:

$$I = -(I_o + I_L)$$

Current due to thermal generation → dark current

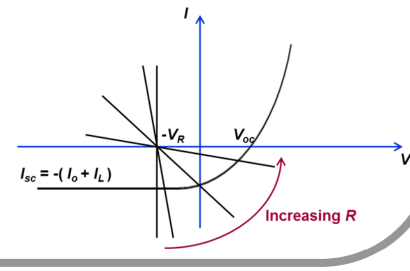


Recall that:

$$I_o \;\alpha\; A n_i^2 \qquad \text{where } n_i^2 = \sqrt{N_c N_v}\, e^{\frac{-E_g}{2kT}}$$

⇒ Small bandgaps and high temperatures increase dark current

Device leakage current also contributes to dark current



Photodetector Bandwidth

We assume that the optical power incident of the photodiode is time dependent:

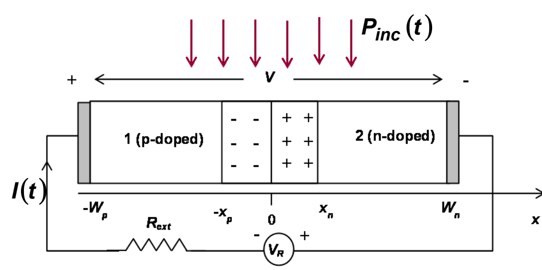
$$P_{inc}(t) = P_o + \text{Re}\left\{ P(f) e^{j2\pi ft} \right\}$$

Suppose the current $I(t)$ in the external circuit is:

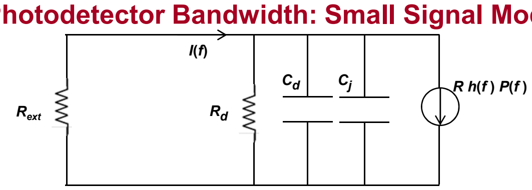
$$I(t) = I_o + \text{Re}\left\{ I(f) e^{j2\pi ft} \right\}$$

We know that: $I_o = R P_o$

But how do we relate $I(f)$ to $P(f)$?



Photodetector Bandwidth: Small Signal Model



R_d = Differential resistance of the pn junction $= \frac{kT}{qI_o} e^{-qV/KT} = \frac{kT}{qI_o} e^{qV_R/KT}$

C_j = Diode junction capacitance

C_d = Diode depletion capacitance due to charge storage in the quasineutral regions and can be ignored in a reverse biased pn junction

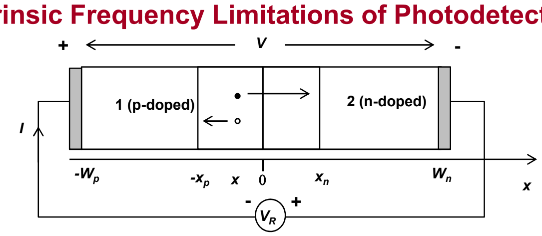
$h(f)$ = A frequency dependent function that describes the intrinsic frequency response of the photogenerated carriers inside the photodiode

$$I(f) = RP(f)h(f)\frac{R_d}{R_d + R_{ext} + j2\pi f\, C_j R_d R_{ext}} \approx RP(f)h(f)\frac{1}{1 + j2\pi f\, C_j R_{ext}}$$

The RC limit of the detector bandwidth

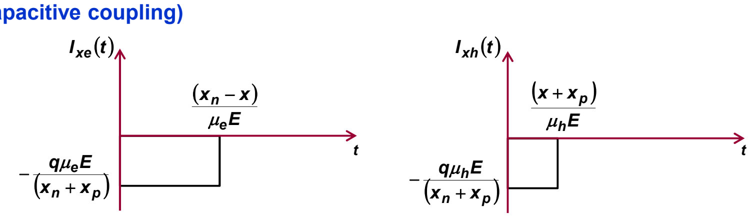
Limiting behavior of $h(f)$: $\begin{cases} h(f \to 0) \to -1 \\ h(f \to \infty) \to 0 \end{cases}$

Intrinsic Frequency Limitations of Photodetectors: $h(f)$



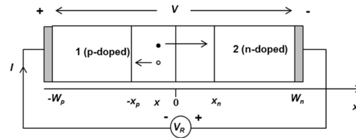
Consider an electron generated at point x inside the junction at time t=0

Ramo-Shockley Theorem:
As the electron and hole move inside the depletion region under the influence of the electric field, there is current flow in the external circuit due to image charges (or capacitive coupling)



$$\int_o^{\infty} [I_{xe}(t) + I_{xh}(t)]\, dt = -q\frac{\mu_e E}{(x_n + x_p)} \cdot \frac{(x_n - x)}{\mu_e E} - q\frac{\mu_h E}{(x_n + x_p)} \cdot \frac{(x + x_p)}{\mu_h E} = -q$$

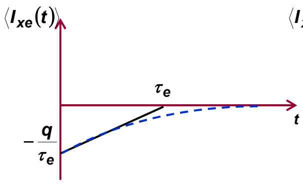
Intrinsic Frequency Limitations of Photodetectors: $h(f)$

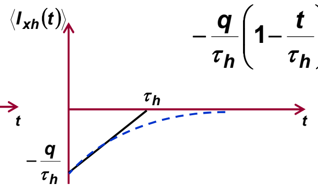


The actual detector current consists of photogenerated pairs at all locations inside the depletion region. So the average current waveform due to one photogeneration event is:

$$\langle I(t)\rangle = \frac{1}{(x_n + x_p)}\int_{-x_p}^{x_n}\left[I_{xe}(t) + I_{xh}(t)\right] dx = -\frac{q}{\tau_e}\left(1 - \frac{t}{\tau_e}\right) \quad \rightarrow 0 \le t \le \tau_e$$

$$-\frac{q}{\tau_h}\left(1 - \frac{t}{\tau_h}\right) \quad \rightarrow 0 \le t \le \tau_h$$





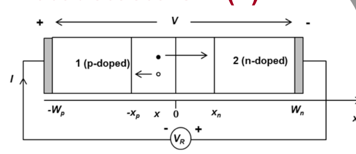
Electron and hole transit times through the entire depletion region are:

$$\tau_e = \frac{(x_n + x_p)}{\mu_e E} \qquad \tau_h = \frac{(x_n + x_p)}{\mu_h E}$$

The above transients can be approximated as:

$$\langle I(t)\rangle = -\frac{q}{2\tau_e} e^{-\frac{t}{\tau_e}} - \frac{q}{2\tau_h} e^{\frac{t}{\tau_h}} \quad (t \ge 0)$$

Intrinsic Frequency Limitations of Photodetectors: $h(f)$



The average current waveform due one photogeneration event is:

$$\langle I(t)\rangle = -\frac{q}{2\tau_e} e^{-\frac{t}{\tau_e}} - \frac{q}{2\tau_h} e^{\frac{t}{\tau_h}} \quad (t \ge 0)$$

This is the impulse response $h(t)$ of the detector: $h(t) = \dfrac{\langle I(t)\rangle}{q}$

The Fourier transform of the impulse response will give $h(f)$:

$$h(f) = -\left[\frac{1/2}{1 + j2\pi f\tau_e} + \frac{1/2}{1 + j2\pi f\tau_h}\right]$$

→ Describes the frequency limitations due to the electron and hole transit times through the depletion region

Finally we have:

$$I(f) = Rh(f)P(f)\frac{1}{1 + j2\pi f\, C_j R_{ext}}$$

$$= -RP(f)\left[\frac{1/2}{1 + j2\pi f\tau_e} + \frac{1/2}{1 + j2\pi f\tau_h}\right]\left[\frac{1}{1 + j2\pi f\, C_j R_{ext}}\right]$$

Describes the frequency limitations due both intrinsic and extrinsic (circuit level) factors

Avalanche Photodiodes (APDs): Basic Principle

Photomultiplier tubes:

- **Provide high sensitivity for light detection even at the single photon level**
- **Employs electron multiplication to increase the charge output per photon**

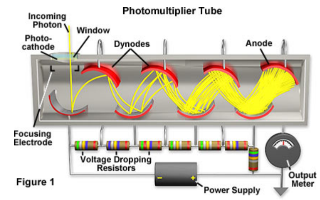
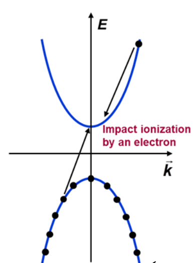
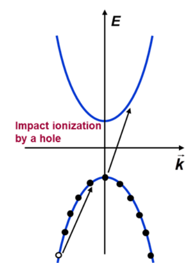


Figure 1

Impact Ionization in Semiconductors:

Highly energetic electrons and holes in semiconductors can create electron-hole pairs via impact ionization

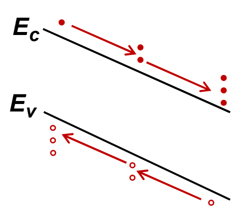


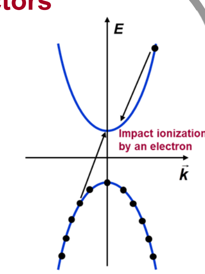


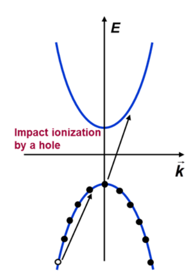
$$E_{min}(\text{electron}) = E_g\left(1 + \frac{m_e}{m_e + m_h}\right) \qquad E_{min}(\text{hole}) = E_g\left(1 + \frac{m_h}{m_e + m_h}\right)$$

Modeling Impact Ionization in Semiconductors

Electrons and holes in high electric fields gain enough energy to cause impact ionization







Impact Ionization Coefficients:

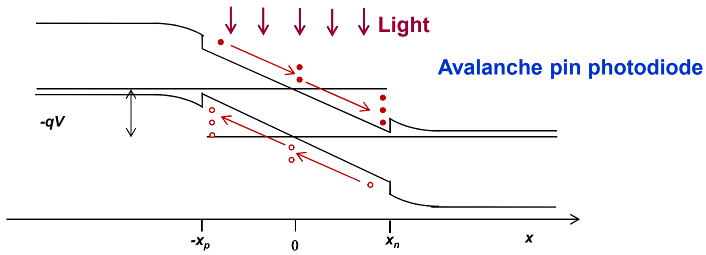
α_e = **The electron ionization coefficient (units: 1/cm) defined as the number of electron-hole pairs created by one electron in unit distance of travel**

α_h = **The hole ionization coefficient (units: 1/cm) defined as the number of electron-hole pairs created by one hole in unit distance of travel**

$$\alpha_e = A_e e^{-C_e / E^{\gamma}}$$

$$\alpha_h = B_h e^{-C_h / E^{\delta}}$$

Avalanche Photodiodes



The equation for the electron current is:

$$-\frac{\partial J_e(x)}{\partial x} = q[G_e(x)+G_L(x)] = \alpha_e |J_e(x)| + \alpha_h |J_h(x)| + qG_L(x)$$

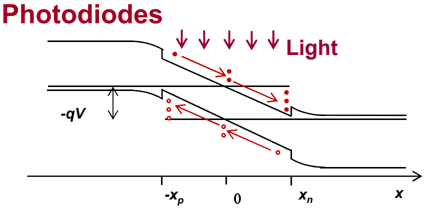
$$\Rightarrow \frac{\partial J_e(x)}{\partial x} = \alpha_e J_e(x) + \alpha_h J_h(x) - qG_L(x)$$

The equation for the hole current is:

$$\frac{\partial J_h(x)}{\partial x} = q[G_h(x)+G_L(x)] = \alpha_e |J_e(x)| + \alpha_h |J_h(x)| + qG_L(x)$$

$$\Rightarrow \frac{\partial J_h(x)}{\partial x} = -\alpha_e J_e(x) - \alpha_h J_h(x) + qG_L(x)$$

Avalanche Photodiodes



Total current is:

$$J_T = J_e(x) + J_h(x)$$

The equation for the electron current becomes:

$$\frac{\partial J_e(x)}{\partial x} - (\alpha_e - \alpha_h) J_e(x) = +\alpha_h J_T - qG_L(x)$$

Solution, assuming uniform illumination, is:

$$J_e(x_n) = J_e(-x_p) e^{(\alpha_e - \alpha_h)(x_n + x_p)} - (qG_L - \alpha_h J_T) \frac{\left[e^{(\alpha_e - \alpha_h)(x_n + x_p)} - 1\right]}{(\alpha_e - \alpha_h)}$$

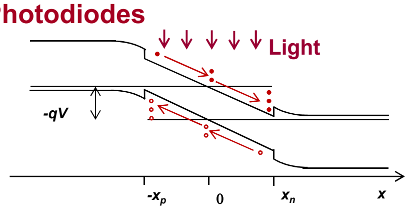
Boundary conditions:

$$J_e(-x_p) = 0 \qquad J_h(x_n) = 0$$

Total current is:

$$J_T = J_e(x_n) + J_h(x_n) = J_e(x_n)$$

Avalanche Photodiodes



Total current is:

$$J_T = J_e(x_n) + J_h(x_n) = J_e(x_n)$$

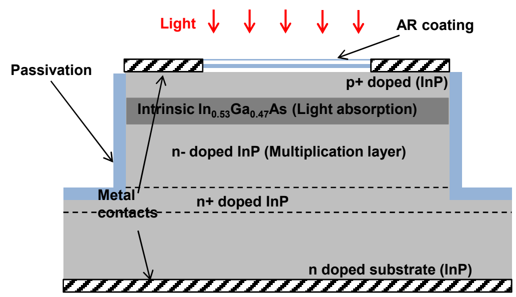
$$\Rightarrow J_T = -qG_L \frac{\left(e^{(\alpha_e-\alpha_h)(x_n+x_p)}-1\right)}{(\alpha_e-\alpha_h)-\alpha_h\left(e^{(\alpha_e-\alpha_h)(x_n+x_p)}-1\right)}$$

The **multiplication gain** ***M*** of the photodetector is:

$$M = \frac{J_T}{qG_L(x_n+x_p)} = \frac{1}{(x_n+x_p)}\frac{\left(e^{(\alpha_e-\alpha_h)(x_n+x_p)}-1\right)}{(\alpha_e-\alpha_h)-\alpha_h\left(e^{(\alpha_e-\alpha_h)(x_n+x_p)}-1\right)}$$

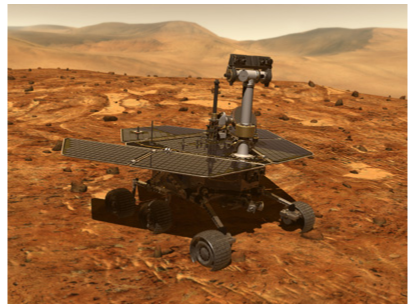
Avalanche Photodiodes

Front side illuminated heterostructure avalanche photodiode with separate photogeneration and multiplication layers



- Avalanche photodiodes are good for high sensitivity but low speed applications
- Most avalanche photodiodes have separate regions for light absorption and carrier multiplication.

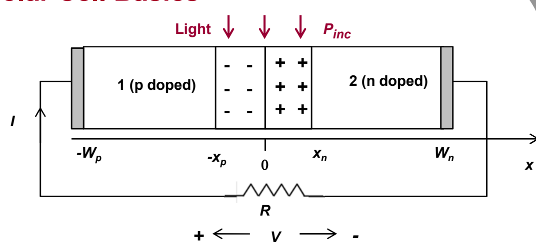
Solar Cell Basics



Solar Cell Basics

$$I = I_o\left(e^{\frac{qV}{KT}} - 1\right) - I_L$$

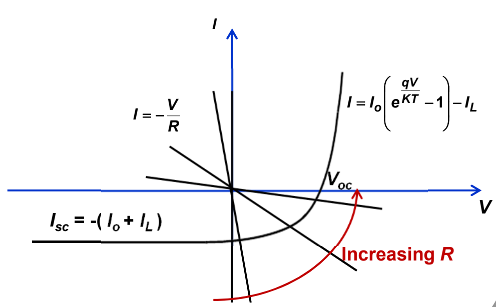
$$IR + V = 0$$



How to deliver the maximum power to the resistor *R*?

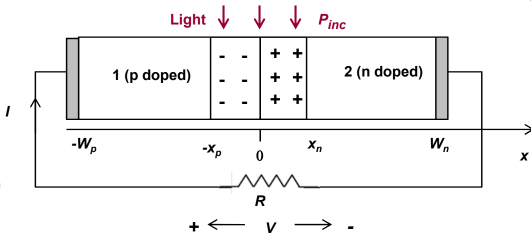
$$P_{out} = \left[I_o\left(e^{\frac{qV}{KT}} - 1\right) - I_L\right](-V)$$

$$\frac{d}{dV}\left\{\left[I_o\left(e^{\frac{qV}{KT}} - 1\right) - I_L\right](-V)\right\} = 0$$



Maximizing Output Power of Solar Cells

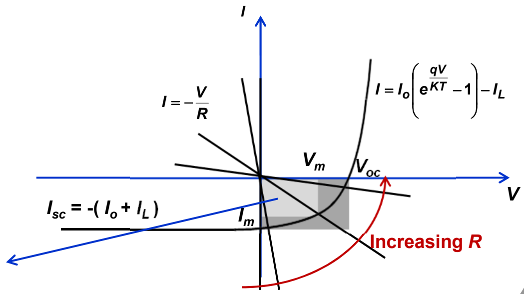
V_m = Voltage at maximum output power
I_m = Current at maximum output power



$$\frac{d}{dV}\left\{\left[I_o\left(e^{\frac{qV}{KT}}-1\right)-I_L\right](-V)\right\}=0$$

$$\Rightarrow e^{\frac{qV_m}{KT}}\left(1+\frac{qV_m}{KT}\right)=\frac{I_L}{I_o}+1$$

$$\Rightarrow I_m = I_o\left(e^{\frac{qV_m}{KT}}-1\right)-I_L$$

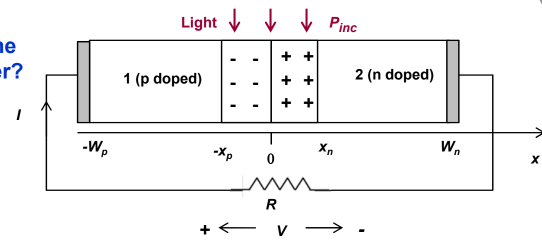


Area of rectangle = $-I_mV_m$ = P_{out} = Power delivered to the resistor R

$$P_{out}=-I_mV_m<-I_{sc}V_{oc}$$

Resistance Matching in Solar Cells

What value of the resistor R lets one achieve the maximum output power?



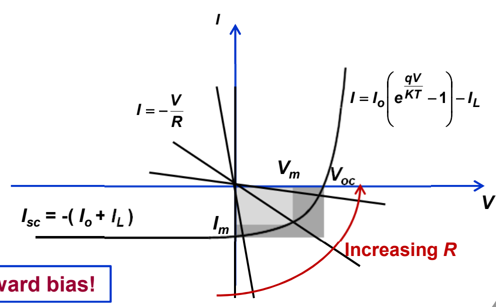
$$\frac{1}{R}=\frac{qI_o}{KT}e^{\frac{qV_m}{KT}}=\frac{1}{R_{dm}}$$

Solar cell Fill Factor (FF):

$$FF=\frac{-I_mV_m}{-I_{sc}V_{oc}}$$

Solar Efficiency:

$$\eta=\frac{-I_mV_m}{P_{inc}}$$



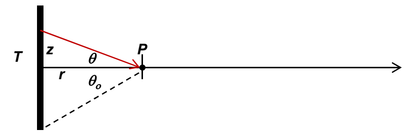
Solar cells are always operated in forward bias!

Black Body Radiation

Solar radiation spectrum near Earth can be very well approximated as that of a black body at temperature T_s = 5760K

Consider a point P at a distance r from a black body at temperature T

We need to calculate the photon flux and the radiation power at the point P



Each mode of radiation emitted from the black body with wavevector $\vec{q}$ will have photon occupancy given by:

$$n(\vec{q}) = \frac{1}{e^{\hbar\omega(\vec{q})/KT} - 1}$$

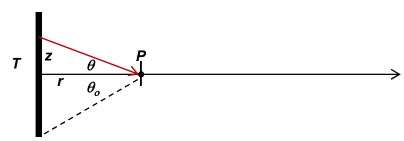
The total photon flux per unit area at P can then be written as an integral:

$$F_N = 2 \times \int_0^{\theta_o} \sin\theta \cos\theta \, d\theta \int_0^{\infty} \frac{2\pi q^2 dq}{(2\pi)^3} \, c \, n(q) = c \, \frac{\sin^2\theta_o}{4} \int_0^{\infty} d\omega \, g_p(\omega) \, n(\omega)$$

Where the photon density of states in free space is:

$$g_p(\omega) = \frac{\omega^2}{\pi^2 c^3}$$

Black Body Radiation

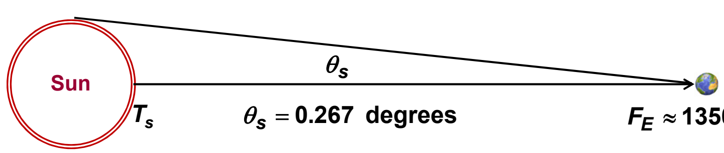


The energy flux at the point P is:

$$F_E = c \, \frac{\sin^2\theta_o}{4} \int_0^{\infty} d\omega \, \hbar\omega \, g_p(\omega) \, n(\omega) = \sin^2\theta_o \, \frac{\pi^2}{60} \frac{(KT)^4}{c^2 \hbar^3} = \sin^2\theta_o \left(\sigma T^4\right)$$

$$\sigma = \text{Stephan} - \text{Boltzmann constant} = 5.67 \times 10^{-8} \, \frac{\text{Watts}}{\text{m}^2\text{-K}^4}$$

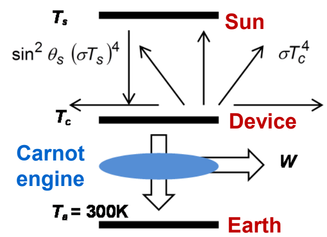
Radiation at the surface of the Earth:



$F_E \approx 1350$ Watts/m^2

Fundamental Limits to Solar Energy Conversion Efficiency

A device at temperature T_c absorbs solar energy, radiates a part of it away, and then converts the rest into useful work W by an ideal heat engine operating between the temperature T_c and the ambient temperature on Earth T_a = 300K



Net radiative energy absorbed by the device (and which is available for useful work):

$$\sin^2\theta_s\left(\sigma T_s^4\right) - \sigma T_c^4$$

Energy conversion efficiency of an ideal heat engine (Carnot engine):

$$\left(1 - T_a/T_c\right)$$

Energy conversion efficiency of the device:

$$\eta = \frac{\left(\sin^2\theta_s\left(\sigma T_s^4\right) - \sigma T_c^4\right)\left(1 - \frac{T_a}{T_c}\right)}{\sin^2\theta_s\left(\sigma T_s^4\right)}$$

But the above expression does not consider the fact that solar energy can be concentrated!

Solar Energy Concentration and Energy Conversion Efficiency

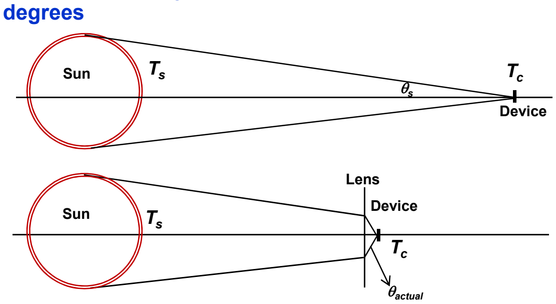
One can increase the efficiency by focusing or concentrating sunlight

Focusing can increase the effective half-angle θ_s of the Sun from 0.267 degrees to the maximum possible value of 90 degrees

Light concentration is measured in units of the parameter X:

$$X = \frac{\sin^2\theta_{actual}}{\sin^2\theta_s}$$

$$1 \le X \le \frac{1}{\sin^2\theta_s} = 4.6\times10^4$$



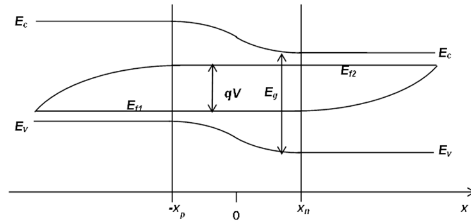
With full concentration, the maximum efficiency for solar energy conversion is:

$$\eta = \frac{\left(\sin^2\theta_s\left(\sigma T_s^4\right) - \sigma T_c^4\right)\left(1 - \frac{T_a}{T_c}\right)}{\sin^2\theta_s\left(\sigma T_s^4\right)} \rightarrow \left(1 - \frac{T_c^4}{T_s^4}\right)\left(1 - \frac{T_a}{T_c}\right)$$

A maximum efficiency of close to 85% is achieved for T_c = 2450K

Fundamental Energy Conversion Efficiency of a Photodiode

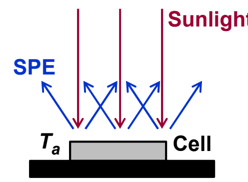
Consider a solar cell made from a semiconductor with a bandgap E_g at temperature T_a = 300K



Assumption: **The solar cells absorbs all photons whose energies are larger than the bandgap**

We know from Chapter 3 that in the spontaneously emitted radiation by a forward bias junction the occupancy of a radiation mode of frequency ω is:

$$n(\omega) = \frac{1}{e^{(\hbar\omega - qV)/KT_a} - 1}$$

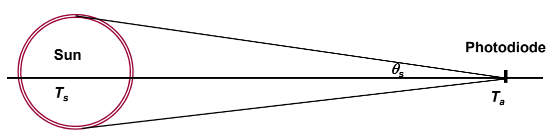


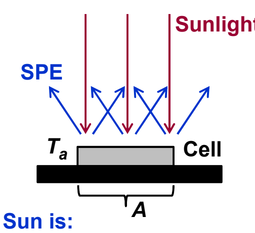
General expressions for emitted photon number and energy fluxes:

$$F_N(T,V,\omega_{\min}) = \frac{c}{4}\int_{\omega_{\min}}^{\infty} d\omega\ g_p(\omega)\frac{1}{e^{(\hbar\omega - qV)/KT} - 1}$$

$$F_E(T,V,\omega_{\min}) = \frac{c}{4}\int_{\omega_{\min}}^{\infty} d\omega\, \hbar\omega\, g_p(\omega)\frac{1}{e^{(\hbar\omega - qV)/KT} - 1}$$

Fundamental Energy Conversion Efficiency of a Photodiode





The radiation power per unit area incident on the cell from the Sun is:

$$A\sin^2\theta_s\ F_E(T_s,0,0) = A\sin^2\theta_s\ \frac{c}{4}\int_{E_g/\hbar}^{\infty} d\omega\ \hbar\omega\, g_p(\omega)\frac{1}{e^{\hbar\omega/KT_s} - 1}$$

The photon flux per unit area incident on the cell from the Sun and absorbed by the cell is:

$$A\sin^2\theta_s\ F_N(T_s,0,E_g/\hbar) = A\sin^2\theta_s\ \frac{c}{4}\int_{E_g/\hbar}^{\infty} d\omega\ g_p(\omega)\frac{1}{e^{\hbar\omega/KT_s} - 1}$$

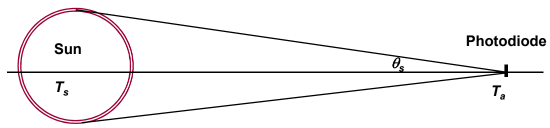
The photon flux per unit area incident on the cell from the surrounding ambient and absorbed by the cell is:

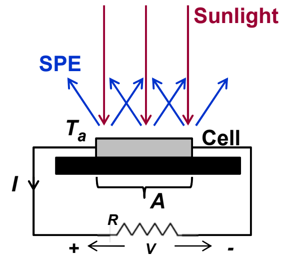
$$A\left(1-\sin^2\theta_s\right)F_N(T_a,0,E_g/\hbar) = A\left(1-\sin^2\theta_s\right)\frac{c}{4}\int_{E_g/\hbar}^{\infty} d\omega\ g_p(\omega)\frac{1}{e^{\hbar\omega/KT_a} - 1}$$

The photon flux per unit area emitted by the cell is:

$$AF_N(T_a,V,E_g/\hbar) = A\frac{c}{4}\int_{E_g/\hbar}^{\infty} d\omega\ g_p(\omega)\frac{1}{e^{(\hbar\omega - qV)/KT_a} - 1}$$

Fundamental Energy Conversion Efficiency of a Photodiode





Assumption: The internal quantum efficiency of the cell is 100% and each photon absorbed by the cell results in one electron flow in the external circuit:

$$I = qA\sin^2\theta_s\, F_N(T_s,0,E_g/\hbar) + A(1-\sin^2\theta_s)F_N(T_a,0,E_g/\hbar) - AF_N(T_a,V,E_g/\hbar)$$

The energy conversion efficiency of the cell is:

$$\eta = \frac{IV}{A\sin^2\theta_s\, F_E(T_s,0,0)}$$

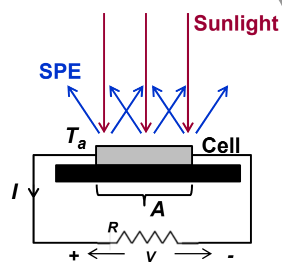
$$= \frac{qA\left[\sin^2\theta_s\, F_N(T_s,0,E_g/\hbar) + (1-\sin^2\theta_s)F_N(T_a,0,E_g/\hbar) - F_N(T_a,V,E_g/\hbar)\right]V}{A\sin^2\theta_s\, F_E(T_s,0,0)}$$

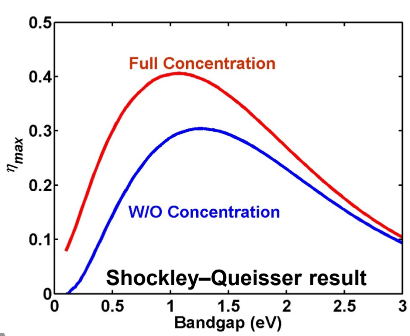
To find the maximum value of η one must maximize the above w.r.t. to the voltage V

Shockley–Queisser Limit

No Concentration (θ_s = 0.26°):
A maximum efficiency of ~31% is reached at a bandgap of ~1.27 eV

Full Concentration (θ_s = 90°):
A maximum efficiency of ~41% is reached at a bandgap of ~1.1 eV.





Why is there an optimal bandgap?

Bandgap too small ⇒ Output voltage too small

$$qV < qV_{oc} < E_g$$

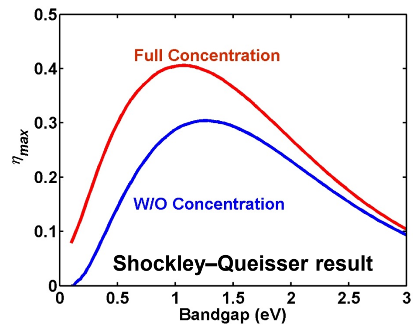
Bandgap too large ⇒ Low energy photons do not get absorbed

Energy Conversion Efficiencies of Some Common Solar Cells

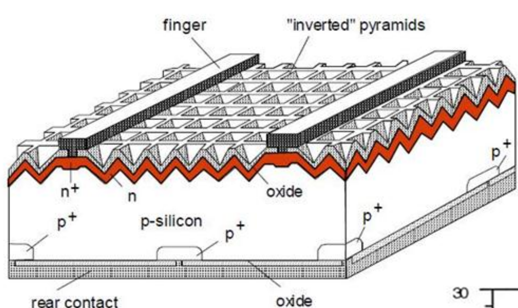
Typical Performances of Semiconductor Photocells

(Green at al., Prog. Photovolt: Res. Appl., 17, 85 (2009))

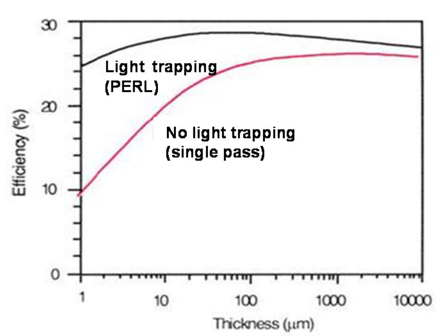
Material	Voc (V)	Jsc (Amp/cm^2)	FF (%)	Efficiency (%)
Crystalline Si	0.705	42.7	82.8	25.0
Crystalline GaAs	1.045	29.7	84.7	26.1
Poly-Si	0.664	38.0	80.9	20.4
a-Si	0.859	17.5	63.0	9.5
CuInGaSe$_2$ (CIGS)	0.716	33.7	80.3	19.4
CdTe	0.845	26.1	75.5	16.7



Design of Silicon Solar Cells

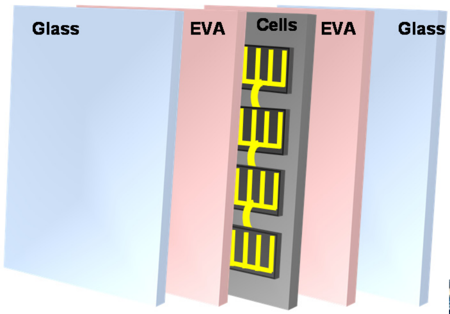


The PERL Si solar cell (Green et al., 1994) Efficiency ~25%



Light trapping by a top lambertian surface and bottom reflector can increase the optical path length by ~4n^2

Design of Solar Cell Modules



A solar cell module

The IV characteristics of pn junctions connected in series must identical (or nearly so)

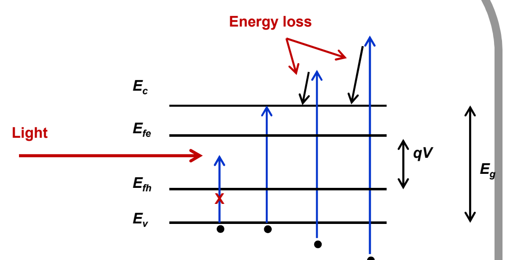
Improving Efficiencies of Photodiode Solar Cells

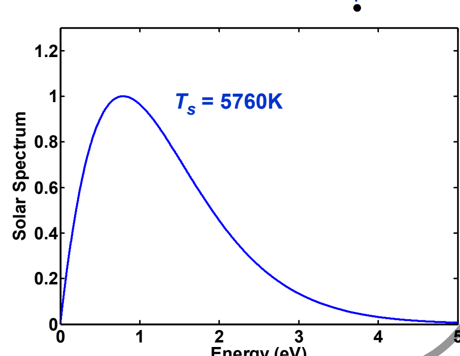
Energy Loss Pathways:

i) Photons with energies smaller than the bandgap are not absorbed

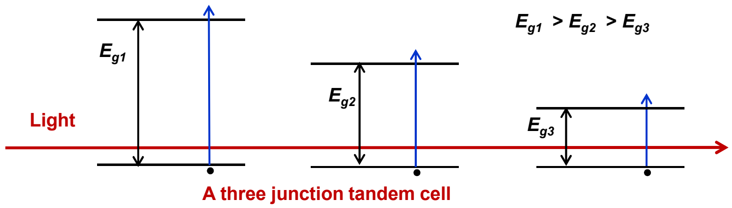
ii) Photons with energies much larger than the bandgap generate electrons (holes) with energies much above (below) the band edge. This extra energy is lost to phonons and does not contribute to output electrical power. Recall that:

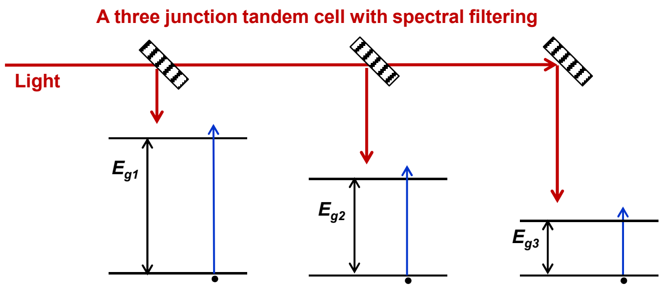
$$qV < qV_{oc} < E_g$$



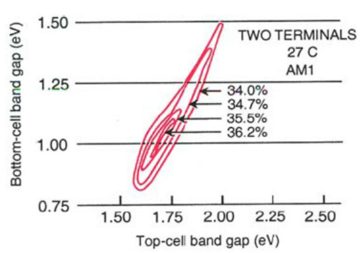


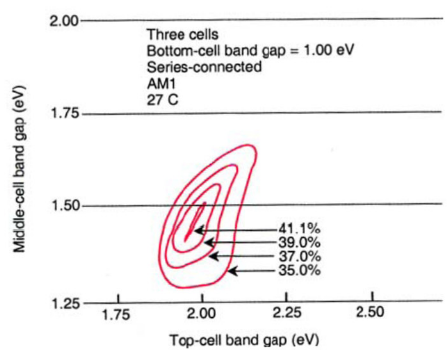
Improving Efficiencies of Solar Cells: Tandem Cells

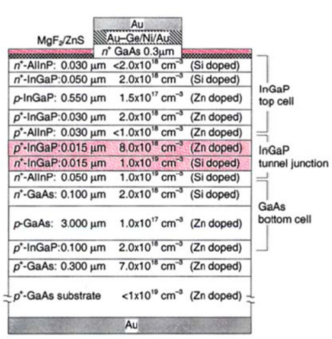




Improving Efficiencies of Solar Cells: Tandem Cells







InGaP/GaAs two-junction tandem cell with a reversed biased Esaki tunnel junction and an efficiency of 30.3% (w/o concentration) (Takamoto et al., 1997).